BT-MRE-I & II/05.14-0426

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B.Tech. Degree I & II Semester Examination in Marine Engineering May 2014

MRE 1101 ENGINEERING MATHEMATICS I

Time: 3 Hours

Maximum Marks: 100

(5 x 20 = 100)

I. (a) Verify Lagrange's Mean value theorem and determine c for $f(x) = x(x-1)(x-2)$ in $(0,1/2)$. (8)

(b) Evaluate $\lim_{x\to 0} \dfrac{(1+x)^{\frac{1}{x}} - e}{x}$ (6)

(c) Evaluate $\lim_{x\to 0}\left(\dfrac{1}{\sin x} - \dfrac{1}{x}\right)$. (6)

OR

II. (a) Show that the radius of curvature at any point of the cardioid $r = a(1-\cos\theta)$ varies as $\sqrt{r}$. (7)

(b) Find the asymptotes of the curve $x^3 + 3x^2y - 4y^3 - x + y + 3 = 0$ (7)

(c) If $y = \sin(m\sin^{-1} x)$, prove that $(1-x^2)y_{n+2} - 2(n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$. (6)

III. (a) If Z is a homogenous function of degree n in x and y, show that (6)

$$x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x\partial y} + y^2\frac{\partial^2 z}{\partial z^2} = n(n-1)z.$$

(b) If $u = xyz,\ v = x^2 + y^2 + z^2,\ w = x + y + z.$ Find $\partial(x,y,z)/\partial(u,v,w)$. (7)

(c) The diameter and altitude of a can in the shape of a right circular cylinder are measured as 4cm and 6cm respectively. The possible error in each measurement 0.1cm. Find approximately the maximum possible error in the values computed for the volume and lateral surface. (7)

OR

IV. (a) Discuss the maxima and minima of $f(x,y) = x^3y^2(1-x-y)$. (6)

(b) Verify Euler's theorem for $\sin^{-1}\dfrac{x}{y} + \tan^{-1}\dfrac{y}{x}$. (6)

(c) Find the volume of the greatest rectangular parallelepiped that can be inscribed in the ellipsoid $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$. (8)

V. (a) Find the equation of hyperbola whose directrix is $2x + y = 1$, focus $(1,2)$ and eccentricity $\sqrt{3}$. (10)

(b) Show that the tangent to a rectangular hypoerbola terminated by its asymptotes is bisected at the point of contact and encloses a triangle of constant area. (10)

OR

(P.T.O.)

VI. (a) Derive the standard equation of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. (8)

(b) Find the equations of the tangents to the ellipse $9x^2+16y^2=144$ from the point $(2,3)$. (6)

(c) Find vertex, focus and directrix of the parabola $4y^2+12x-12y+39=0$ (6)

VII. (a) Find the reduction formula for $\int x^n \sin mx\, dx$. (6)

(b) Find the area enclosed by the curve $a^2x^2=y^3(2a-y)$. (7)

(c) Find the entire length of the cardiod $r=a(1-\cos\theta)$. (7)

OR

VIII. (a) Evaluate $\iint r^3 dr d\theta$ over the area included between the circles $r=2\sin\theta$ and $r=4\sin c\theta$. (7)

(b) Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dz dy dx$. (6)

(c) Calculate by double integration, the volume bounded by the cylinder $x^2+y^2=4$ and the planes $y+z=4$ *and* $z=0$. (7)

IX. (a) Show that the volume of the tetrahedron ABCD is $\frac{1}{6}\left[A\vec{B},A\vec{C},A\vec{D}\right]$. (10)

(b) Prove that $(\vec{B}\times\vec{C}).(\vec{A}\times\vec{D})+(\vec{C}\times\vec{A}).(B\times\vec{D})+(\vec{A}\times\vec{B}).(\vec{C}\times\vec{D})=0$ (10)

OR

X. (a) Prove that $\nabla^2 f(r)=f''(r)+\frac{2}{r}f'(r)$. (7)

(b) Prove that curl $(grad\phi)=0$. (6)

(c) Prove that $\vec{F}=(y^2\cos x+z^3)i+(2y\sin x-4)j+3xz^2k$ is irrotational and find its scalar potential. (7)

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